

# Introduction to Microfluidics

2010/03/05

Yi-Chung Tung, Ph.D.

Research Center for Applied Sciences

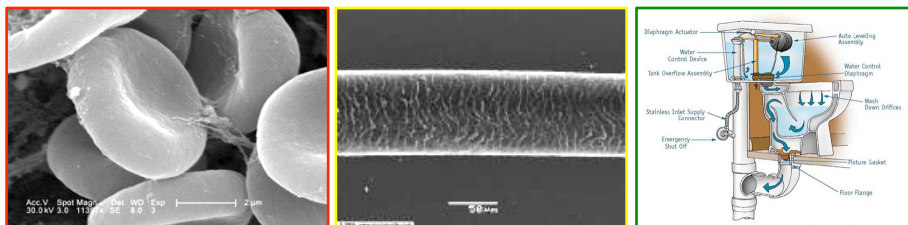
Academia Sinica, Taipei, Taiwan

## Outline

- Introduction to Microfluidics
- Basic Fluid Mechanics Concepts
- Equivalent Fluidic Circuit Model
- Conclusion

## What is Microfluidics ?

- **Microfluidics** = Micro + Fluidics
- **Micro**:  $10^{-6}$ 
  - Small size (sub-mm)
  - Small volumes ( $\mu\text{l}$ , nl, pl)
- **Fluidics**: handling of liquids and/or gases



## Advantages of Microfluidics

- The motivation for using a microfluidic system is analogous to the argument for using integrated circuits (IC) to replace the discrete component circuits.
- Advantages:

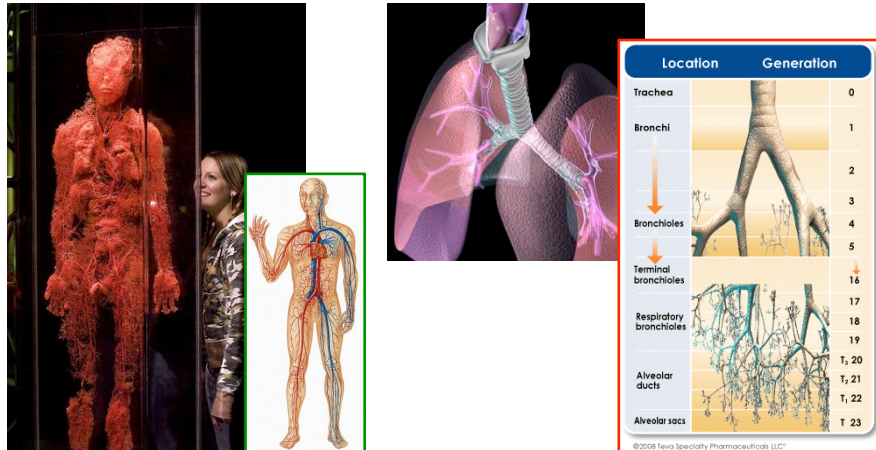
Miniaturization: Portability (Lab-on-a-Chip)

Integration: Low Costs, Batch Fabrication

Automation: Simplicity of Operation

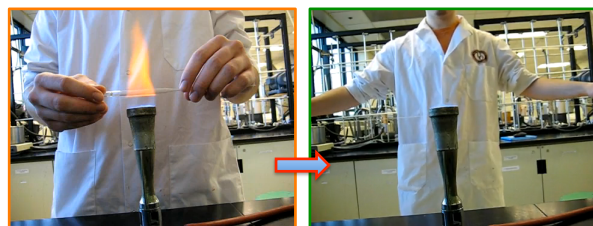
## Microfluidics *in vivo*

- Circulating and Respiratory System



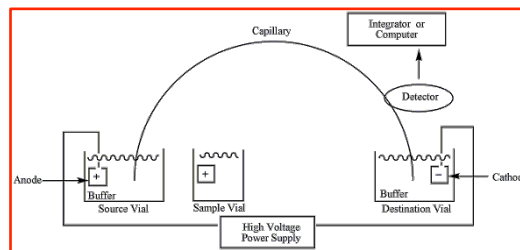
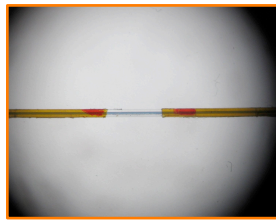
## Old School Microfluidics

- Historical Microfluidics: Glass capillary
- Glass Capillary has been broadly exploited in labs
- Capillary Electrophoresis (CE)



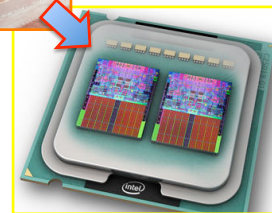
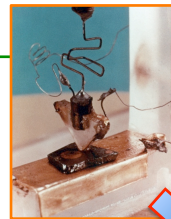
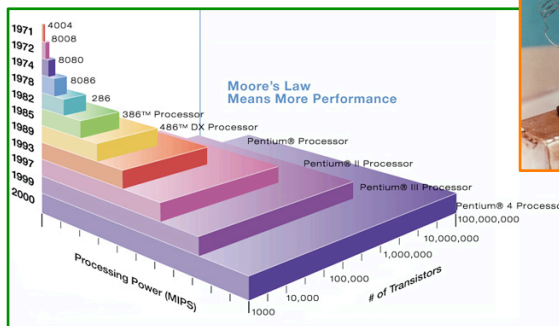
## Capillary Electrophoresis (CE)

- Introduced in the 1960s, the technique of capillary electrophoresis (CE) was designed to separate species based on their size to charge ratio in the interior of a small capillary filled with an electrolyte.



## New Era of Microfluidics

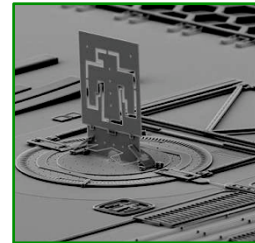
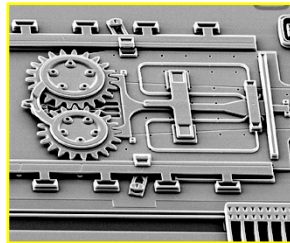
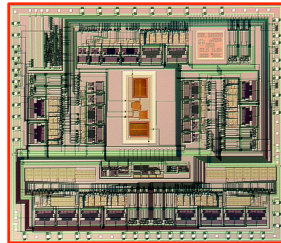
- Due to the advancement of micro/nano fabrication technology, “microfluidics” has been redefined.



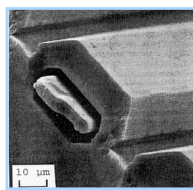


## Microelectromechanical Systems

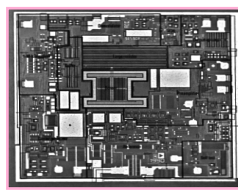
- Microelectromechanical Systems (MEMS).
- MEMS is the integration of *mechanical elements, sensors, actuators, and electronics* on a common silicon substrate through microfabrication technology.



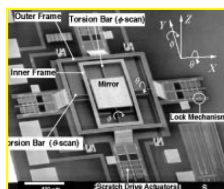
## Advancement of MEMS



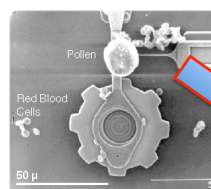
Ikeda et al. 1990 (Japan)



Analog Device ADXL50

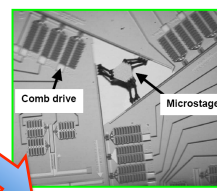


Toshiyoshi et al., 1998 (UCLA)

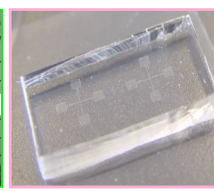


mems.andia.gov

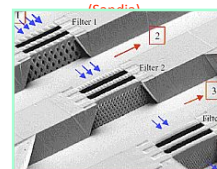
Polymer



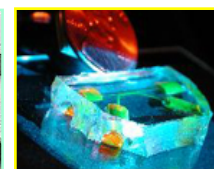
Walraven et al., 2003



www.uni-ulm.de



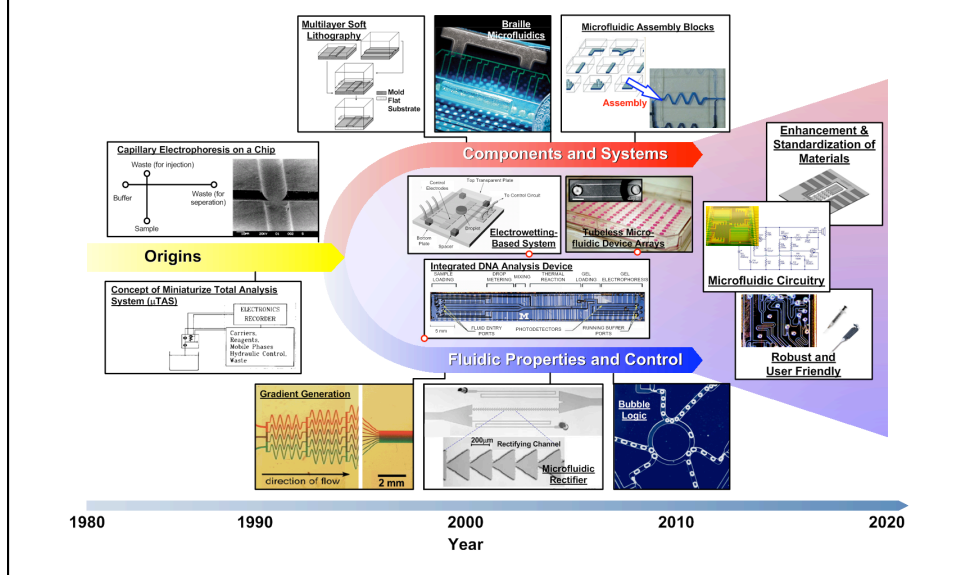
www.mic.dtu.dk



Takayama et al, 2003 (UM)

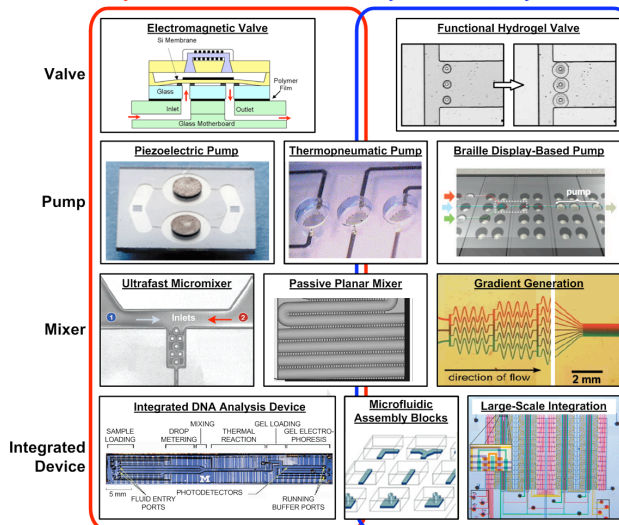
Silicon-Based

## Development of Microfluidics



## Materials of Microfluidic Devices

Crystalline Si, Quartz, and Glass Polymers: PDMS, Polycarbonate...



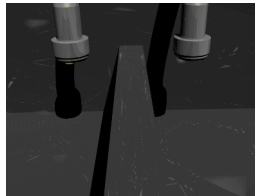
## Fluid Mechanics

- **Density ( $\rho$ ):**

Density = Mass/Volume ( $\text{kg/m}^3$ )

- **Dynamic Viscosity (Viscosity,  $\mu$ ):**

Viscosity is a measure of the resistance of a fluid which is being deformed by either shear stress or tensional stress.



Temp (°C)	Density (kg/m <sup>3</sup> )
100	958.4
80	971.8
60	983.2
40	992.2
30	995.6502
25	997.0479
22	997.7735
20	998.2071
15	999.1026
10	999.7026
4	999.9720
0	999.8395
-10	998.117
-20	993.547
-30	983.854

The density of water in kilograms per cubic meter (SI unit) at various temperatures in degrees Celsius.  
The values below 0 °C refer to [supercooled water](#).

## Fluid Mechanics

- **Viscosity ( $\mu$ ) – Newton's Theory:**

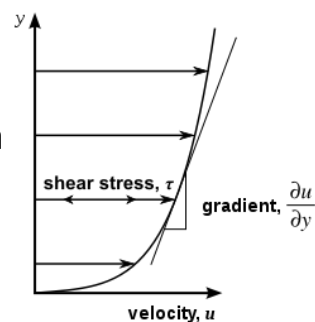
$$\tau = \mu \frac{du}{dy}$$

where  $\tau$  is Fluid Shear Stress

where  $(du/dy)$  is change of fluid velocity in  $y$  direction

- This is a constitutive equation (a reasonable first approximation).

- **Newtonian Fluids.**



## Fluid Mechanics

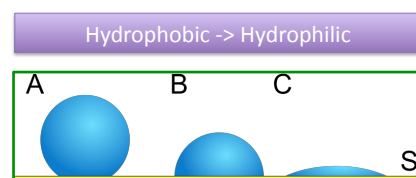
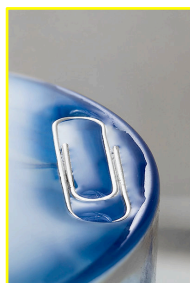
- **Viscosity ( $\mu$ ):**
  - Unit:  $\text{kg}/(\text{m}\cdot\text{s}) = \text{Pa}\cdot\text{s} = 10 \text{ Poise}$
  - Viscosity of Water at  $20^\circ\text{C}$ 
    - =  $0.01002 \text{ Poise}$
    - =  $1.002 \text{ cP}$  (centipoise)
    - =  $1.002 \text{ mPa}\cdot\text{s}$
  - Viscosity of SAE 30 oil at  $20^\circ\text{C}$ 
    - =  $2.9 \text{ Poise}$

Temperature [ $^\circ\text{C}$ ]	Viscosity [ $\text{mPa}\cdot\text{s}$ ]
10	1.308
20	1.002
30	0.7978
40	0.6531
50	0.5471
60	0.4668
70	0.4044
80	0.3550
90	0.3150
100	0.2822

## Fluid Mechanics

- **Surface Tension ( $\gamma$ ):**

**Surface tension** is a property of the surface of a liquid. Surface tension is caused by cohesion (the attraction of molecules to like molecules).

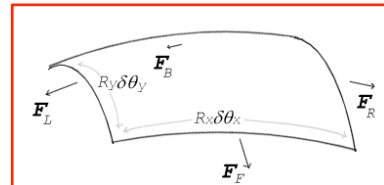


## Fluid Mechanics

- Surface Tension ( $\gamma$ ):**

**Young-Laplace Equation:**

$$\Delta p = \gamma \left( \frac{1}{R_x} + \frac{1}{R_y} \right)$$



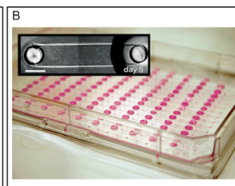
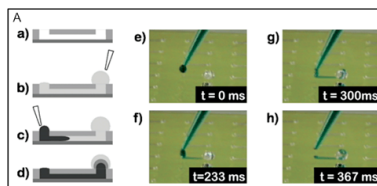
Where  $\Delta p$  is the pressure difference,  $\gamma$  is surface tension,  $R_x$  and  $R_y$  are radii of curvature in each of the axes that are parallel to the surface.

Water at 25°C,  $\gamma = 71.97$  (dyn/cm)

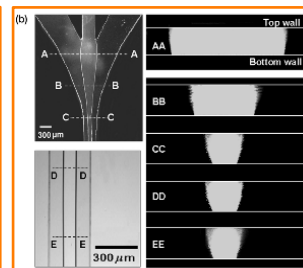
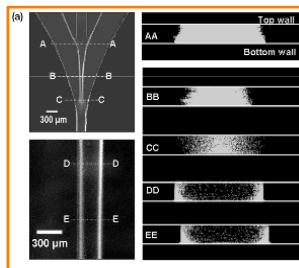
$\Delta p$ for water drops of different radii at STP				
Droplet radius	1 mm	0.1 mm	1 $\mu\text{m}$	10 nm
$\Delta p$ (atm)	0.0014	0.0144	1.436	143.6

## Fluid Mechanics

- Utilize Surface Tension in Microfluidics**



Surface Tension  
Driven Flow



Air-Liquid Two  
Phase Flow



## Fluid Mechanics

- **Reynolds Number ( $Re$ ):**

$Re$  is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces.

$$Re = \frac{\rho VL}{\mu}$$

$V$  is the mean fluid velocity (m/s)

$L$  is the a characteristic linear dimension (m)  
(traveled length of fluid or hydarulic diameter etc.)

## Fluid Mechanics

- **Reynolds Number ( $Re$ ):**

- Large  $Re$  means inertial force dominates: **Turbulent Flow** (unsteady flow stream, i.e. swirls and vortices).

- Small  $Re$  means viscosity force dominates: **Laminar Flow** (fluid stream follows regular paths, i.e. streamlines).

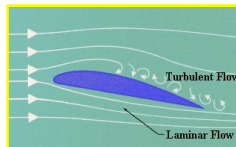
- For flow in a pipe, laminar flow occurs when  $Re < 2300$ , and turbulent flow occurs when  $Re > 4000$ .

- In the interval between 2300 and 4000: **Transition Flow**.

## Fluid Mechanics

- **Laminar Flow vs. Turbulent Flow**

- Blood Flow in brain  $\sim 1 \times 10^2$
- Blood flow in aorta  $\sim 1 \times 10^3$
- Typical pitch in Major League Baseball  $\sim 2 \times 10^5$
- Person swimming  $\sim 4 \times 10^6$



Low Speed  
Laminar  
small  $Re$



High Speed  
Turbulent  
high  $Re$

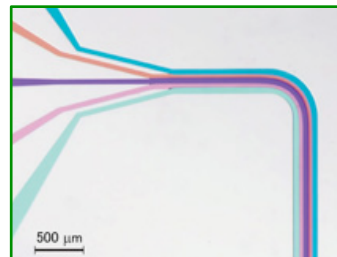
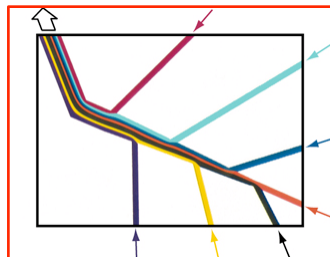
## Fluid Mechanics

- **Microfluidics (Laminar or Turbulent?)**

$$Re = \frac{\rho VL}{\mu}$$

For example:  $V = 1 \text{ (mm/s)}$ ,  $L = 100 \text{ }\mu\text{m}$

$$Re = \frac{\rho VL}{\mu} = \frac{1000(\text{kg/m}^3) \times 10^{-3}(\text{m/s}) \times 100 \cdot 10^{-6}(\text{m})}{0.001(\text{kg/m} \cdot \text{s})} = 0.1$$

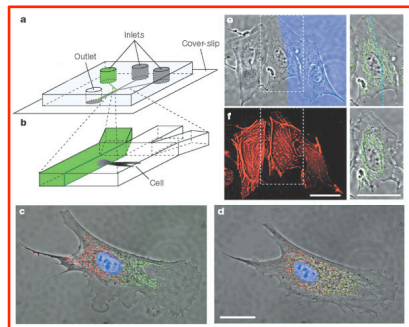




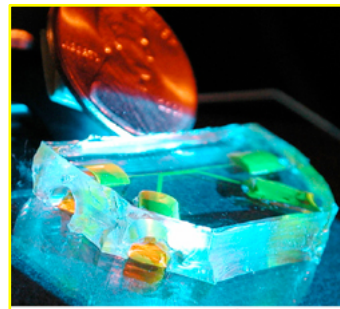
## Fluid Mechanics

- Laminar Flow in Microfluidic Channel

Partial Treatment of Cells using Laminar Flows  
(PARTCELL)



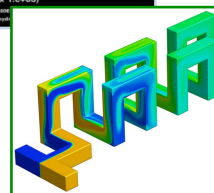
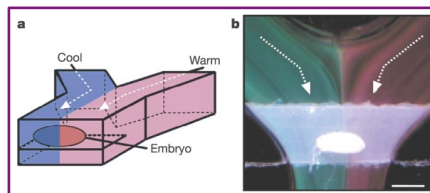
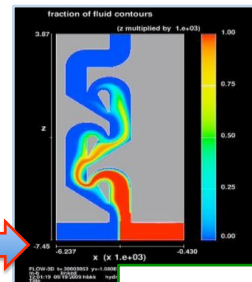
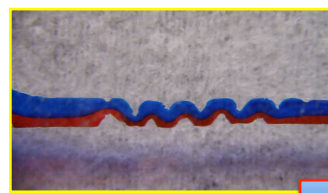
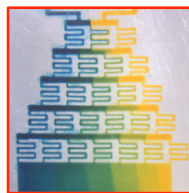
Sperm Sorter  
(Laminar vs. Diffusion)



## Fluid Mechanics

- Limits in Microfluidic Channel

Gradients Generation – Difficult to Mix



## Fluid Mechanics

- **Reynolds Number ( $Re$ ):**

The ratio of inertial forces to viscous forces.

$$Re = \frac{\rho V L}{\mu}$$

- **Weber Number ( $We$ ):**

The ratio of inertia forces to surface tension forces.

$$We = \frac{\rho V^2 L}{\sigma}$$

- **Bond Number ( $Bo$ ):**

The ratio of body forces to surface tension forces.

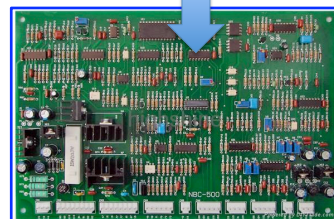
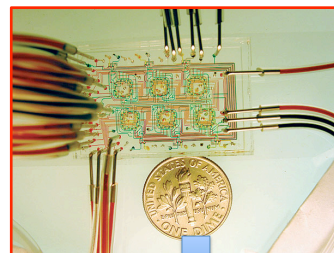
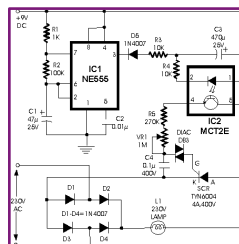
$$Bo = \frac{\rho a L^2}{\gamma}$$

## Equivalent Fluidic Circuit Model

- **Equivalent Circuit Model:**

Voltage -> Pressure

Current -> Flow Rate



## Equivalent Fluidic Circuit Model

- **Equivalent Circuit Model (Resistance, R):**

Analogous to electrical resistance, fluid resistance is defined as the ratio of pressure drop over flow rate,

$$R = \frac{\Delta P}{Q} \text{ in } \frac{N \cdot s}{m^5}$$

where  $\Delta P$  is the pressure difference, in  $N/m^2$ , and  $Q$  is the volume flow rate, in  $m^3/s$ .

For a pipe with a rectangular cross section with width  $w$ , and depth  $h$ , and assuming both, laminar flow and Newtonian fluid, the resistance is

$$R = \frac{12\mu L}{w \cdot h^3} \left[ 1 - \frac{h}{w} \left( \frac{192}{\pi^5} \sum_{n=1}^{\infty} \frac{1}{n^5} \tanh\left(\frac{n\pi w}{h}\right) \right) \right]^{-1}$$

## Equivalent Fluidic Circuit Model

- **Equivalent Circuit Model (Capacitance, C):**

Compliant elements of a fluidic system exhibit the fluidic equivalent of capacitance as a pressure-dependent volume change

$$C = \frac{dV}{dP} \text{ in } \frac{m^5}{N}$$

The fluidic capacitance for a square membrane can be derived by plate theory as

$$C = \frac{6a^6(1-\nu^2)}{\pi^4 E t^3}$$

where  $a$  is membrane width, in  $m$ ,  $E$  is Young's modulus of membrane, in  $N/m^2$ ,  $t$  is membrane thickness, in  $m$ , and  $\nu$  is Poisson's ratio of membrane (dimensionless.)

## Equivalent Fluidic Circuit Model

- **Equivalent Circuit Model (Inductance, H):**

In a manner analogous to electrical inductance, fluidic systems are capable of storing kinetic energy in fluidic inductance,  $H$  (in  $\text{kg/m}^4$ )

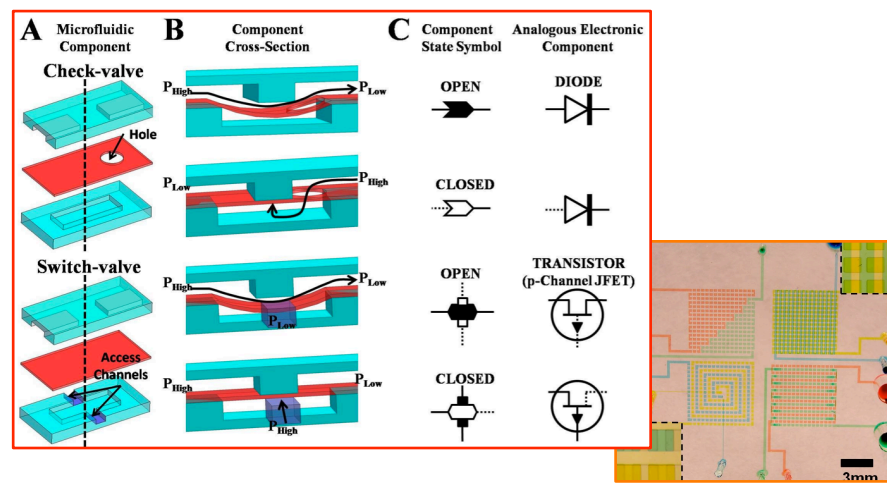
$$\Delta P = H \frac{dQ}{dt}$$

For incompressible and inert fluidics in tubes of constant cross section  $A$ , the fluidic inductance is given by

$$H = \frac{\rho L}{A}$$

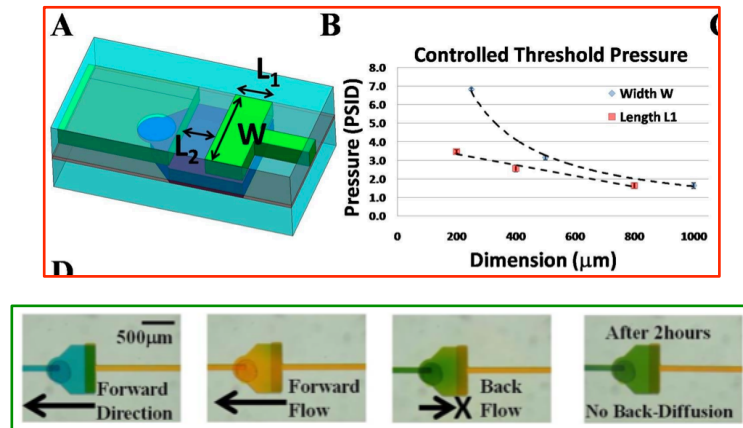
## Equivalent Fluidic Circuit Model

- **Integrated Microfluidic Circuitry Device:**



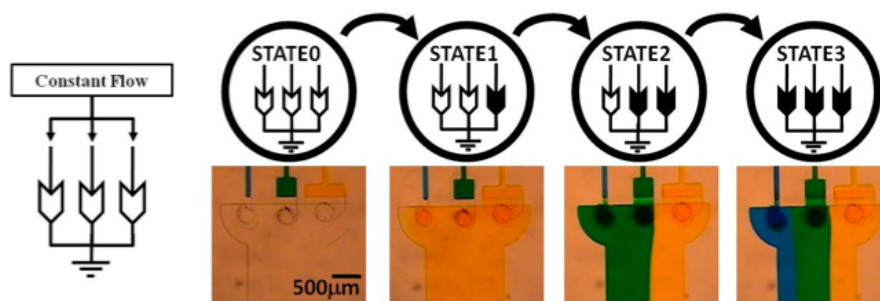
## Equivalent Fluidic Circuit Model

- Integrated Microfluidic Circuitry Device:



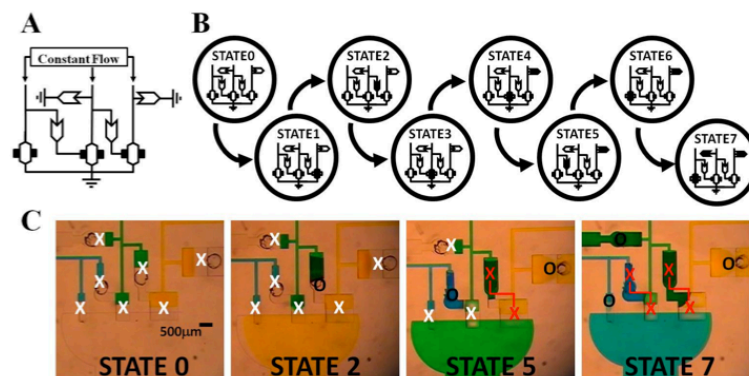
## Equivalent Fluidic Circuit Model

- Integrated Microfluidic Circuitry Device –  
Time Release



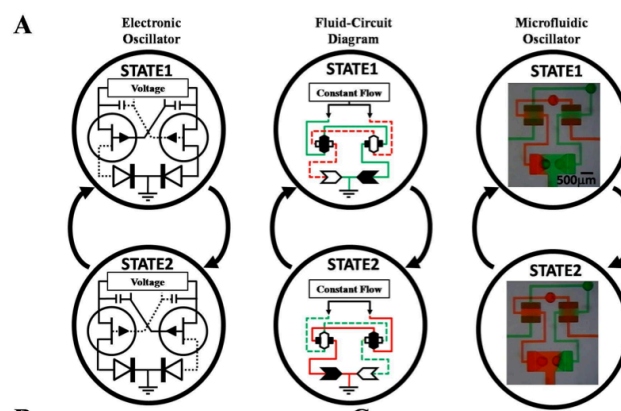
## Equivalent Fluidic Circuit Model

- **Integrated Microfluidic Circuitry Device – Time Switch**



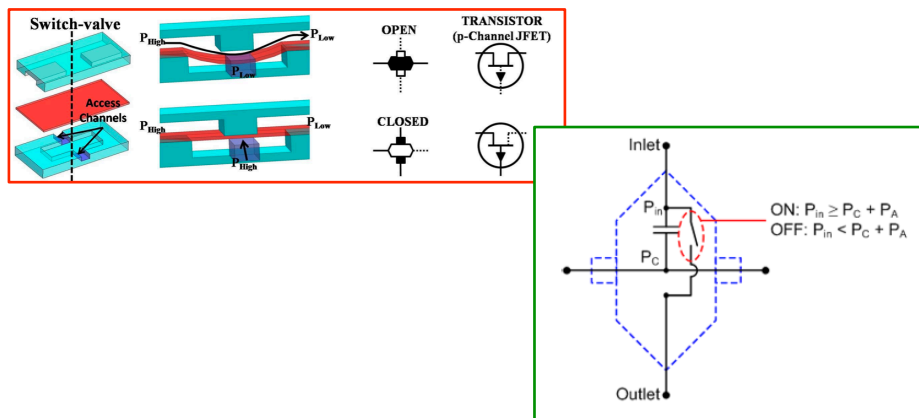
## Equivalent Fluidic Circuit Model

- **Integrated Microfluidic Circuitry Device – Oscillator**



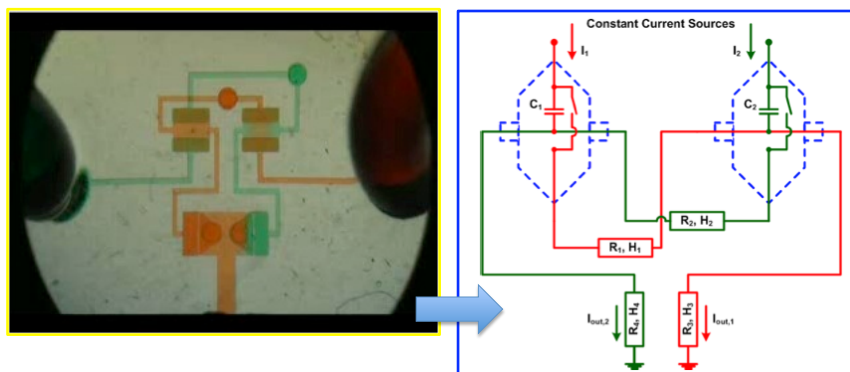
## Equivalent Fluidic Circuit Model

- **Integrated Microfluidic Circuitry Device – Oscillator**



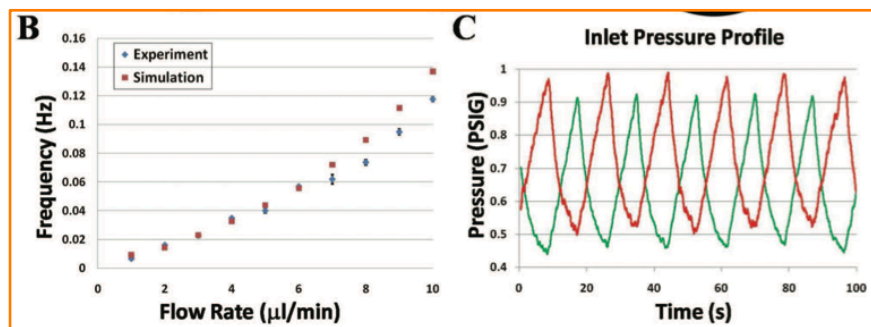
## Equivalent Fluidic Circuit Model

- **Integrated Microfluidic Circuitry Device – Oscillator**



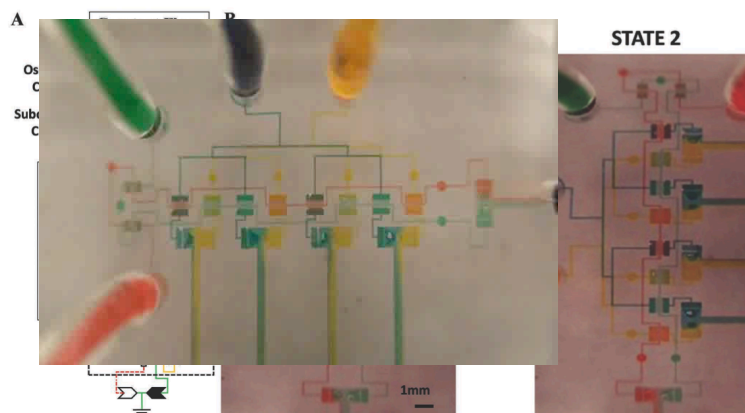
## Equivalent Fluidic Circuit Model

- **Integrated Microfluidic Circuitry Device – Oscillator**



## Equivalent Fluidic Circuit Model

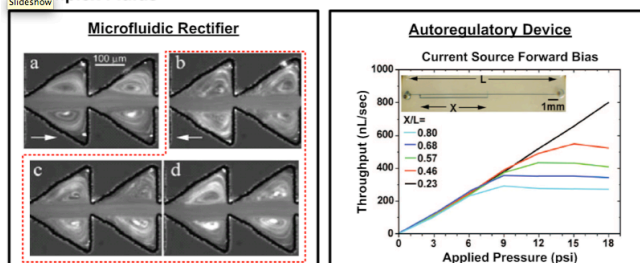
- **Integrated Microfluidic Circuitry Device – Oscillator**



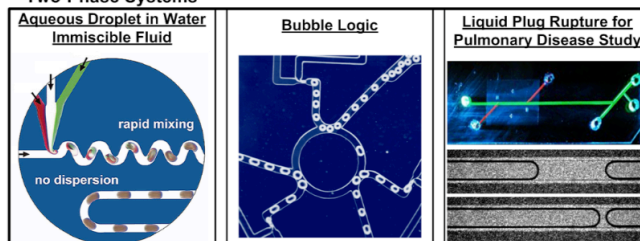


## Other Microfluidic Devices

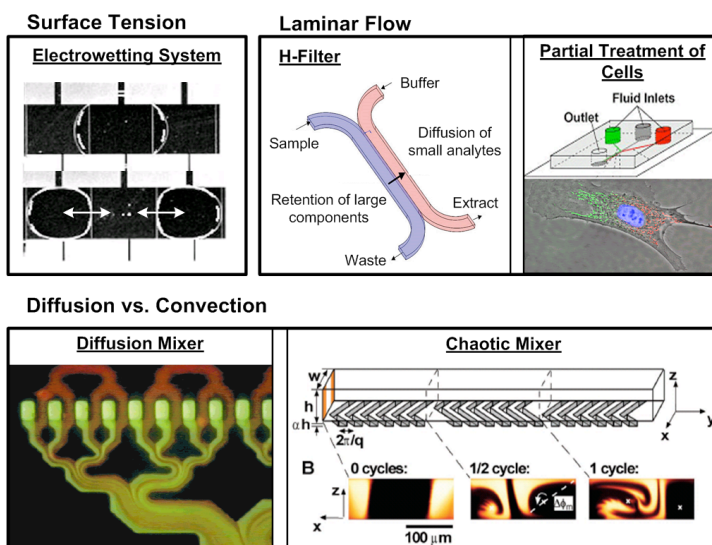
### Complex Fluids



### Two-Phase Systems



## Conclusion



## Conclusion

- Advantages and Limitations of Microfluidics
- Challenges
- Future Directions

